AC MGFs Q&A 1: 0 bits in 00-free bit strings

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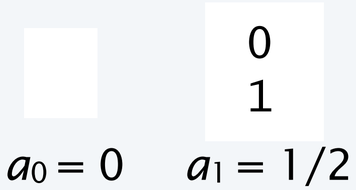

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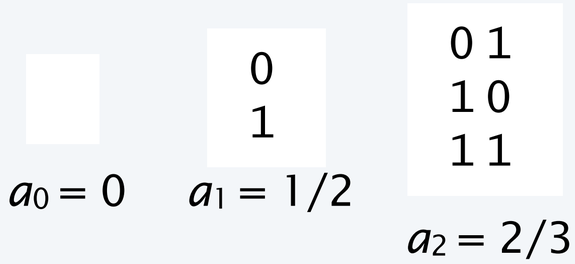

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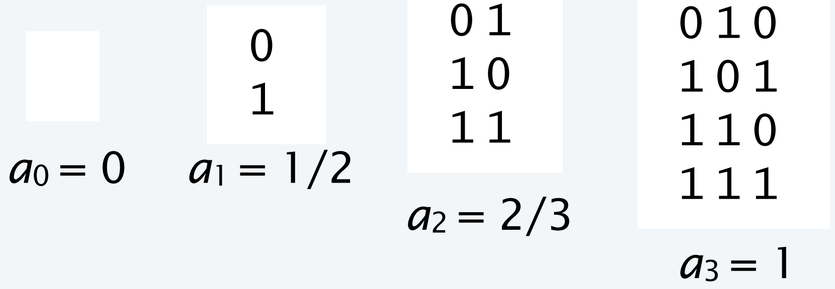

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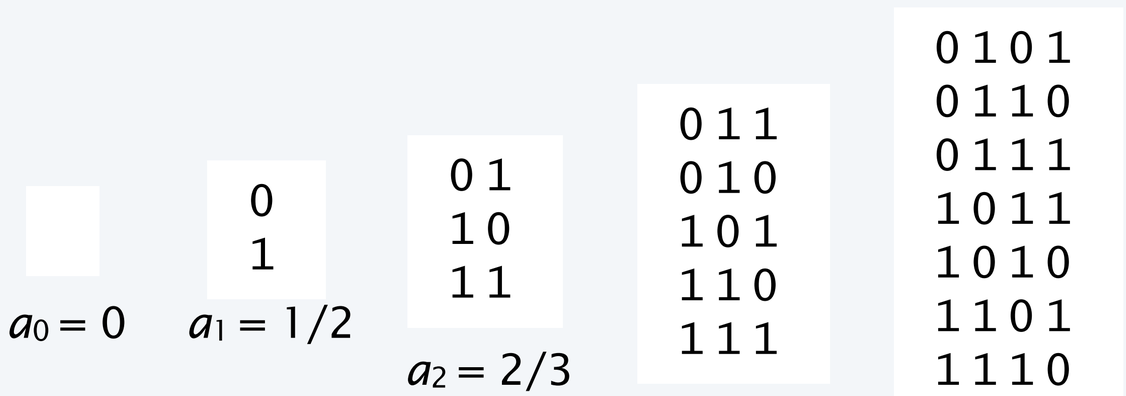

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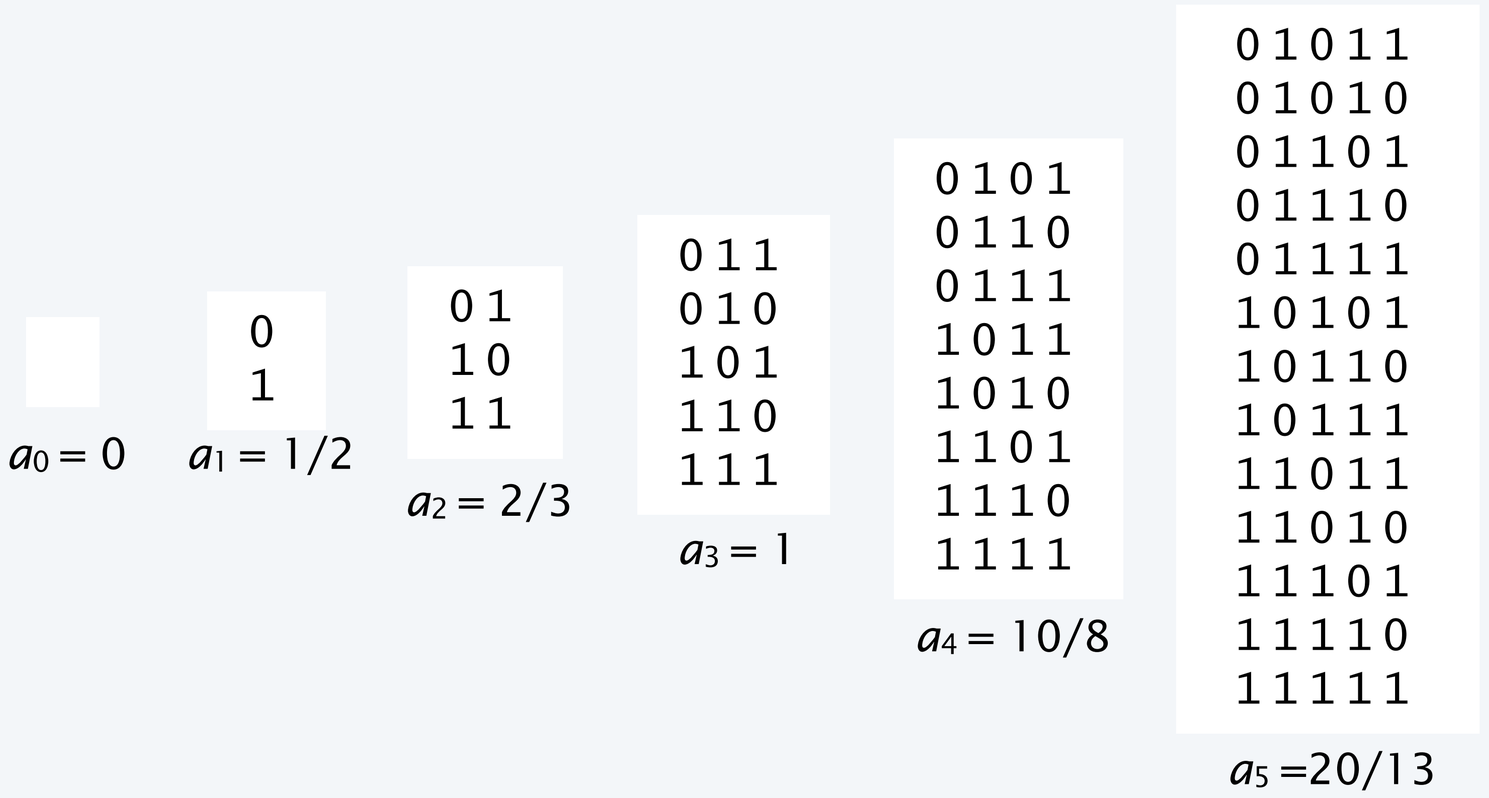

AC MGFs Q&A 1: 0 bits in 00-free bit strings

Q. Recalling the derivation at left, fill in the boxes at right to prove that the *average number of 0 bits in a random bit string of length n containing no 00* is $\sim \frac{n}{\phi\sqrt{5}}$

construction $B = E + Z_0 + (Z_1 + Z_0 \times Z_1) \times B$

OGF equation $B(z) = 1 + z + (z + z^2)B(z)$

BGF equation $B(z, u) =$

OGF $B(z) = \frac{1+z}{1-z-z^2}$

BGF

enumeration $[z^n]B(z) \sim \frac{1+1/\phi}{1-\hat{\phi}/\phi}\phi^n = \frac{\phi^{n+2}}{\sqrt{5}}$

cumulated cost $[z^n]B_u(z, 1) =$

$$[z^n]\frac{f(z)}{(1-z/\rho)^\alpha} \sim \frac{f(\rho)}{\Gamma(\alpha)}\rho^{-n}n^{\alpha-1}$$

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cumulated cost

$$[z^n]B_u(z, 1) = [z^n]\frac{z}{(1 - z - z^2)^2}$$

$$\sim \frac{1/\phi}{(1 - \hat{\phi}/\phi)^2} n\phi^n = n\frac{\phi^{n+1}}{5}$$

$$[z^n]\frac{f(z)}{(1 - z/\rho)^\alpha} \sim \frac{f(\rho)}{\Gamma(\alpha)}\rho^{-n}n^{\alpha-1}$$

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AC MGFs Q&A 2

Q. Which of the following is true of the average number of 0s in a random bit string of length n excluding *any* fixed pattern p, as n grows?

Approaches 0

Approaches 1

Approaches cn for some fixed constant c

Has periodic behavior

Asymptotic order of growth depends on the pattern

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Approaches 0	**F**
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Approaches cn for some fixed constant c	
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Approaches 1	**F**
Approaches cn for some fixed constant c	**T**
Has periodic behavior	**F**
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Has periodic behavior	**F**
Asymptotic order of growth depends on the pattern	**F**