

## AC MGFs Q&A 1: 0 bits in 00-free bit strings

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Q. What is the average number of 0 bits in a random bit string of length  $n$  containing no 00?

$$a_0 = 0 \quad a_1 = \frac{1}{2}$$

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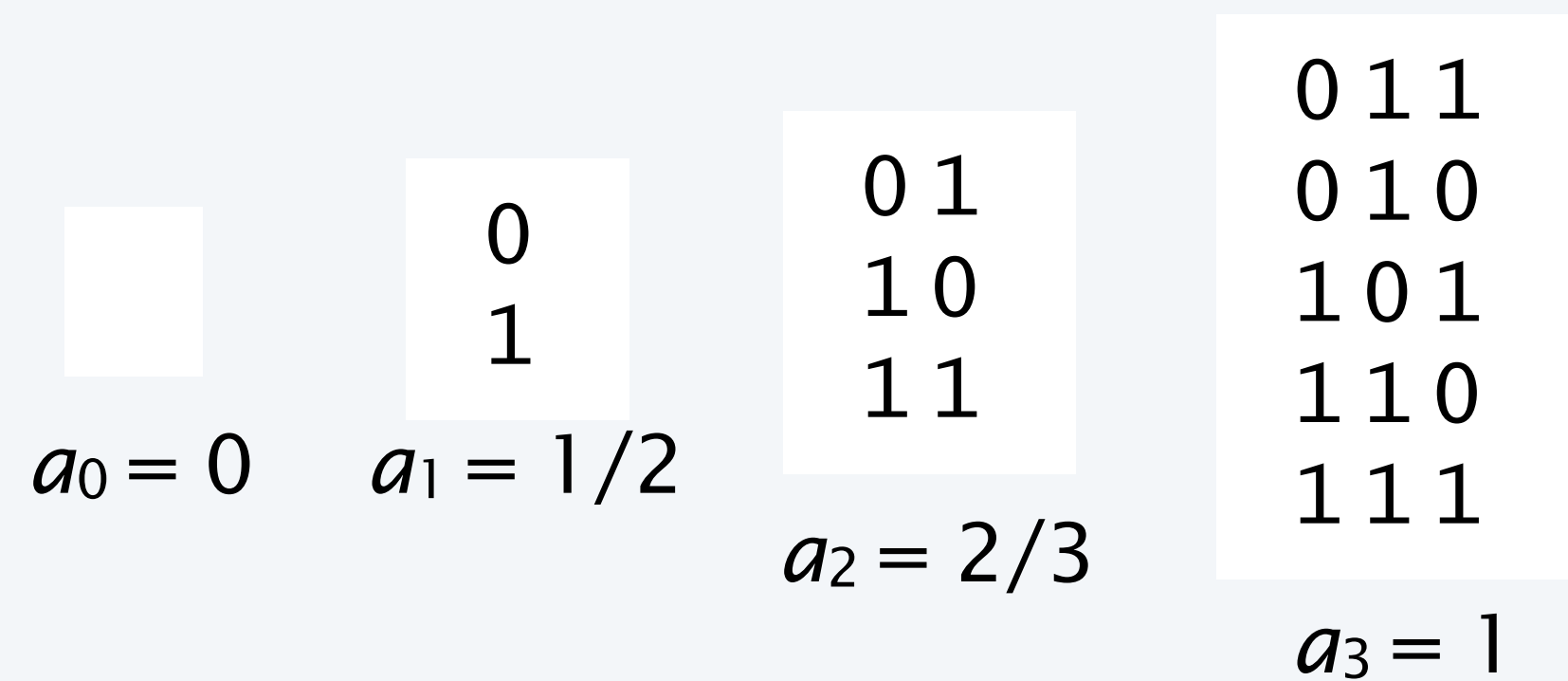
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$\square$	$\begin{array}{c} 0 \\ 1 \end{array}$	$\begin{array}{c} 01 \\ 10 \\ 11 \end{array}$
$a_0 = 0$	$a_1 = 1/2$	$a_2 = 2/3$

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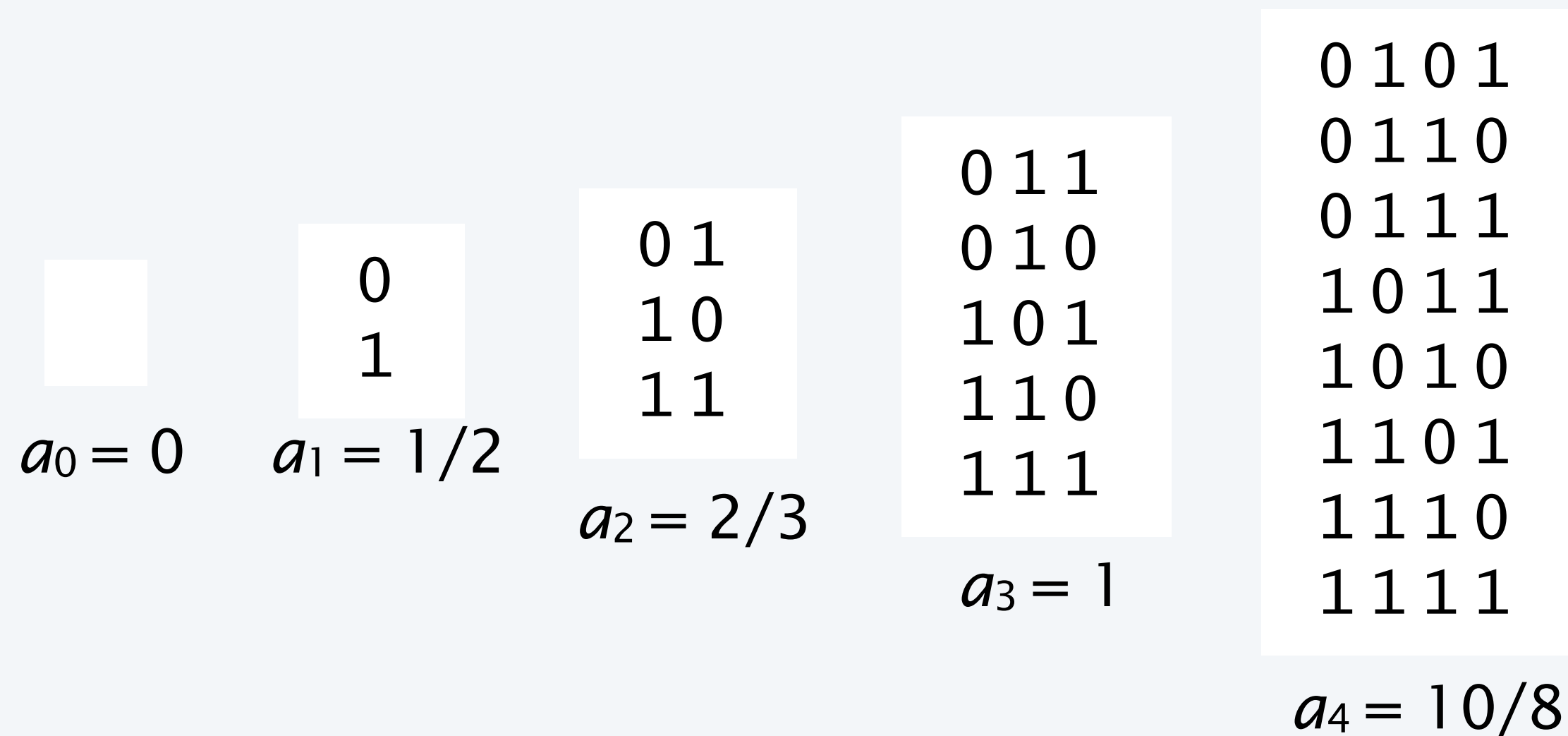
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$\square$

$$a_0 = 0$$

0  
1

$$a_1 = 1/2$$

0 1  
1 0  
1 1

$$a_2 = 2/3$$

0 1 1  
0 1 0  
1 0 1  
1 1 0  
1 1 1

$$a_3 = 1$$

0 1 0 1  
0 1 1 0  
0 1 1 1  
1 0 1 1  
1 0 1 0  
1 1 0 1  
1 1 1 0  
1 1 1 1

$$a_4 = 10/8$$

0 1 0 1 1  
0 1 0 1 0  
0 1 1 0 1  
0 1 1 1 0  
0 1 1 1 1  
1 0 1 0 1  
1 0 1 1 0  
1 0 1 1 1  
1 1 0 1 1  
1 1 0 1 0  
1 1 1 0 1  
1 1 1 1 0  
1 1 1 1 1

$$a_5 = 20/13$$

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Q. Recalling the derivation at left, fill in the boxes at right to prove that the average number of 0 bits in a random bit string of length  $n$  containing no 00 is  $\sim \frac{n}{\phi\sqrt{5}}$

**construction**  $B = E + Z_0 + (Z_1 + Z_0 \times Z_1) \times B$

**OGF equation**  $B(z) = 1 + z + (z + z^2)B(z)$

**OGF**  $B(z) = \frac{1+z}{1-z-z^2}$

**enumeration**  $[z^n]B(z) \sim \frac{1+1/\phi}{1-\hat{\phi}/\phi} \phi^n = \frac{\phi^{n+2}}{\sqrt{5}}$

**BGF equation**  $B(z, u) =$

**BGF**

**cumulated cost**  $[z^n]B_u(z, 1) =$

$$[z^n] \frac{f(z)}{(1-z/\rho)^\alpha} \sim \frac{f(\rho)}{\Gamma(\alpha)} \rho^{-n} n^{\alpha-1}$$

**solution**  $\frac{[z^n]B_u(z, 1)}{[z^n]B(z)} \sim \frac{n}{\phi\sqrt{5}}$

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 $\sim \frac{1/\phi}{(1 - \hat{\phi}/\phi)^2} n\phi^n = n \frac{\phi^{n+1}}{5}$

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Q. Which of the following is true of the average number of 0s in a random bit string of length  $n$  excluding *any* fixed pattern  $p$ , as  $n$  grows?

Approaches 0

Approaches 1

Approaches  $cn$  for some fixed constant  $c$

Has periodic behavior

Asymptotic order of growth depends on the pattern

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## AC MGFs Q&A 2

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Q. Which of the following is true of the average number of 0s in a random bit string of length  $n$  excluding *any* fixed pattern  $p$ , as  $n$  grows?

Approaches 0

**F**

Approaches 1

**F**

Approaches  $cn$  for some fixed constant  $c$

Has periodic behavior

**F**

Asymptotic order of growth depends on the pattern

## AC MGFs Q&A 2

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Approaches 0

**F**

Approaches 1

**F**

Approaches  $cn$  for some fixed constant  $c$

**T**

Has periodic behavior

**F**

Asymptotic order of growth depends on the pattern

## AC MGFs Q&A 2

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Approaches 0

**F**

Approaches 1

**F**

Approaches  $cn$  for some fixed constant  $c$

**T**

Has periodic behavior

**F**

Asymptotic order of growth depends on the pattern

**F**