Q. How many ways to make change for $N$ cents using only nickels, dimes and quarters?

$$Q = \text{MSET}(Z^5 + Z^{10} + Z^{25})$$

$$Q(z) = \frac{1}{(1 - z^5)(1 - z^{10})(1 - z^{25})}$$

$$[z^N]Q(z) \sim \frac{N^2}{5 \cdot 10 \cdot 25 \cdot 2!} = \frac{N^2}{2500}$$

$f(z)$ rational with a single dominant pole $\alpha$

$$[z^N]f(z) = \frac{\beta^N N^{M-1}}{(M-1)! \alpha^M} \lim_{z \to \alpha} (z - \alpha)^M f(z)$$

where $\beta = 1/\alpha$ and $M$ is the multiplicity of $\alpha$
AC Pole Apps Q&A: Compositions with restrictions

**Q.** How many ways to write \( N \) as an ordered sum of (positive) odd integers?

\[
C_1 = 1 \\
C_2 = 1 \\
C_3 = 2 \\
C_4 = 3 \\
C_5 = 5
\]

\[
C = \text{SEQ}( z + z^3 + z^5 + z^7 + ... )
\]

\[
C'(z) = \frac{1}{1 - z - z^3 - z^5 - ...} = \frac{1}{1 - z (1 + z^2 + z^4 + ...)} = \frac{1}{1 - \frac{z}{1-z^2}} = \frac{1 - z^2}{1 - z - z^2}.
\]

**Exercise.** Direct proof that it is a Fibonacci sequence?
AC Pole Apps Q&A: “Black and white reversible strings”

Q. Asymptotics of \([z^N]\) in \[G(z) = \frac{z(1-3z^2)}{(1-2z)(1-2z^2)}\]

A. \(2^{N-2}\)

\(f(z)\) rational with a single dominant pole \(\alpha\)

\([z^N]f(z) = \frac{\beta^NN!}{(M-1)!\alpha^M} \lim_{z \to \alpha} (z-\alpha)^M f(z)\)

where \(\beta = 1/\alpha\) and \(M\) is the multiplicity of \(\alpha\)

**Exercise.** Prove that \(G(z)\) is the OGF for “black-and-white reversible strings”