Analytic transfer theorems ("common cases")

Rational functions.

Meromorphic functions.

Standard function scale.

Supercritical sequences.

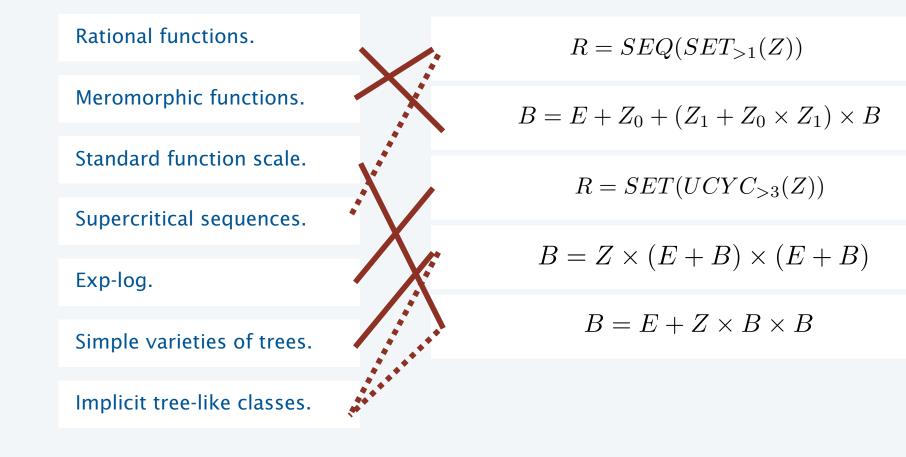
Set schema (exp-log).

Simple varieties of trees.

Implicit tree-like classes.

Analytic transfer theorems ("common cases")

Q. Match each construction with the analytic transfer theorem best suited to solving it.



Q. A "simple variety of trees"

construction
$$B = z \times (1 + B) \times (1 + B)$$

OGF equation $B(z) = z(1 + B(z))^2$

simple variety of trees

Theorem. If a simple variety of trees with GF $F(z) = z\phi(F(z))$ is λ -invertible (where λ is the positive real root of $\phi(u) = u\phi'(u)$) $[z^{N}]F(z) \sim \frac{1}{\sqrt{2\pi\phi''(\lambda)/\phi(\lambda)}} \left(\phi'(\lambda)\right)^{N} N^{-3/2}$ then

characteristic equat

OGF equation

acteristic equation
$$1+2u+u^2=2u+2u^2$$
 solution $\lambda=1$ 4^n

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 $\phi(u) = (1+u)^2$ $\phi'(u) = 2 + 2u$ $\phi''(u) = 2$

binary trees with n internal nodes (Catalan)

~-approximation

3

AC SA Apps Q&A: 3-ary trees

Q. How many 3-ary trees with *n* internal nodes?

construction
$$B = Z \times (1+B)^3$$
OGF equation $B(z) = z(1+B(z))^3$ characteristic
equation $(1+u)^3 = 3u(1+u)^2$ solution $\lambda = 1/2$ ~-approximation $\sim \frac{(27/4)^n}{\sqrt{224}}$

 $\sqrt{8\pi n^3/3}$

simple variety of trees

Theorem. If a simple variety of trees with GF $F(z) = z\phi(F(z))$ is λ -invertible (where λ is the positive real root of $\phi(u) = u \phi'(u)$) $[z^{N}]F(z) \sim \frac{1}{\sqrt{2\pi\phi''(\lambda)/\phi(\lambda)}} \left(\phi'(\lambda)\right)^{N} N^{-3/2}$ then

$$\phi(u) = (1+u)^3$$

$$\phi'(u) = 3(1+u)^2$$

$$\phi''(u) = 6(1+u)$$

4

AC SA Apps Q&A: Motzkin trees with a restriction

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Def. A *skinny Motzkin tree* is an ordered, rooted, unlabelled tree whose node degrees are all 0, 1, or 2, with the restriction that the left child of every 2-node is either a leaf or a 1-node.

Q. Show that the number of skinny Motzkin trees is $\ \sim c \phi^{2N} N^{-3/2}$ for some constant c.

construction	$A = Z + Z \times A + Z \times (Z + Z \times A) \times A$	
OGF equation	$A(z) = z + zA(z) + z^{2}A(z) + z^{2}A(z)^{2}$	
	$\Phi(z, w) = z + zw + z^2w + z^2w^2 = w$	implicit tree-like classes
characteristic system		
	$\Phi_w(z, w) = z + z^2 + 2z^2w = 1$	
solution	$z = 1/\phi^2$ $w = \phi$	
-approximation	$\sim c\phi^{2N} N^{-3/2}$	