Analytic transfer theorems ("common cases")

Rational functions.

Meromorphic functions.

Standard function scale.

Supercritical sequences.

Set schema (exp-log).

Simple varieties of trees.

Implicit tree-like classes.

Analytic transfer theorems ("common cases")

Q. Match each construction with the analytic transfer theorem best suited to solving it.

- Rational functions.
- Meromorphic functions.
- Standard function scale.
- Supercritical sequences.
- Exp-log.
- Simple varieties of trees.
- Implicit tree-like classes.

$$R = SEQ(SET_{>1}(Z))$$

$$B = E + Z_0 + (Z_1 + Z_0 \times Z_1) \times B$$

$$R = SET(UCYC_{>3}(Z))$$

$$B = Z \times (E + B) \times (E + B)$$

$$B = E + Z \times B \times B$$

AC SA Apps Q&A: a "simple variety of trees"

Q. A "simple variety of trees"

construction $B = z \times (1 + B) \times (1 + B)$

OGF equation $B(z) = z(1 + B(z))^2$

characteristic equation $1 + 2u + u^2 = 2u + 2u^2$

solution $\lambda = 1$

~-approximation $\sim \dfrac{4^n}{\sqrt{\pi n^3}}$

binary trees with n internal nodes (Catalan)

simple variety of trees

Theorem. If a simple variety of trees with GF $F(z) = z\phi(F(z))$ is λ-invertible (where λ is the positive real root of $\phi(u) = u\phi'(u)$) then $[z^N]F(z) \sim \dfrac{1}{\sqrt{2\pi\phi''(\lambda)/\phi(\lambda)}}\big(\phi'(\lambda)\big)^N N^{-3/2}$

$$\phi(u) = (1 + u)^2$$
$$\phi'(u) = 2 + 2u$$
$$\phi''(u) = 2$$

AC SA Apps Q&A: 3-ary trees

Q. How many *3*-ary trees with *n* internal nodes?

construction $B = Z \times (1 + B)^3$

OGF equation $B(z) = z(1 + B(z))^3$

characteristic equation $(1 + u)^3 = 3u(1 + u)^2$

solution $\lambda = 1/2$

~-approximation $\sim \dfrac{(27/4)^n}{\sqrt{8\pi n^3/3}}$

simple variety of trees

Theorem. If a simple variety of trees with GF $F(z) = z\phi(F(z))$ is λ-invertible (where λ is the positive real root of $\phi(u) = u\phi'(u)$)

then $[z^N]F(z) \sim \dfrac{1}{\sqrt{2\pi\phi''(\lambda)/\phi(\lambda)}}\left(\phi'(\lambda)\right)^N N^{-3/2}$

$$\phi(u) = (1 + u)^3$$
$$\phi'(u) = 3(1 + u)^2$$
$$\phi''(u) = 6(1 + u)$$

AC SA Apps Q&A: Motzkin trees with a restriction

Def. A *skinny Motzkin tree* is an ordered, rooted, unlabelled tree whose node degrees are all 0, 1, or 2, with the restriction that the left child of every 2-node is either a leaf or a 1-node.

Q. Show that the number of skinny Motzkin trees is $\sim c\phi^{2N}N^{-3/2}$ for some constant *c*.

construction $$A = Z + Z \times A + Z \times (Z + Z \times A) \times A$$

OGF equation $$A(z) = z + zA(z) + z^2A(z) + z^2A(z)^2$$

characteristic system
$$\Phi(z,w) = z + zw + z^2w + z^2w^2 = w$$
$$\Phi_w(z,w) = z + z^2 + 2z^2w = 1$$

solution $z = 1/\phi^2 \quad w = \phi$

~-approximation $\sim c\phi^{2N}N^{-3/2}$

implicit tree-like classes