

# Guidelines for preparing Questions and Answers

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## “Questions and Answers” (Q&As)

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One topic of class meetings for COS 488 is to develop good questions for future exams.

### Properties of a good exam question.

- Easy to understand.
- Easy to grade.
- Solvable in 10 minutes or less (but not trivial).
- Tests understanding of an important topic.
- “Fair” (no tricks)
- Teaches something (optional but desirable)



↑  
Your grade will be based on these criteria!

For examples, see Q&A from *Analysis of Algorithms* (selected examples to follow).

# Easy to understand

## AofA GFs Q&A 1

Q. Match each of the following sequences with their OGF.

0, 0, 1, 3, 6, 10, ...

0, 0, 1/2, 0, 1/4, 0, 1/6, ...

1, 3, 9, 27, 81, 243, ...

1, 1 + 1/2, 1 + 1/2 + 1/3, ...

3, 3, 3, 3, 3, ...

$$\frac{1}{1-3z}$$

$$\frac{z^2}{(1-z)^3}$$

$$\ln \frac{1}{1-z^2}$$

$$\frac{3}{1-z}$$

$$\ln \frac{1}{1-2z}$$

$$\frac{1}{1-z} \ln \frac{1}{1-z}$$

$$\frac{1}{(1-z)^3}$$

## Easy to grade

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### AofA GFs Q&A 2

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Q. Suppose that  $a_n$  satisfies  $a_n = 9a_{n-1} - 20a_{n-2}$  for  $n > 1$  with  $a_0 = 0$  and  $a_1 = 1$

What is  $\lim_{n \rightarrow \infty} a_n/a_{n+1}$  ?

A. **5**

$$a(z) = \frac{z}{1 - 9z + 20z^2} = \frac{z}{(1 - 4z)(1 - 5z)} = \frac{1}{1 - 5z} - \frac{1}{1 - 4z}$$

$$a_n = 5^n - 4^n$$

## Solvable in 10 minutes or less (but not trivial)

### AofA Analytic Combinatorics Q&A 1 (improved version)

Q. Match each combinatorial class with a construction.

binary strings

binary strings with no 01

binary strings with no 11

binary strings with no 001

binary strings with no 00

$$E + (Z_0 + Z_1) \times B = B + (Z_0 \times Z_1) \times B$$

$$B = E + Z_1 + (Z_0 + Z_1 \times Z_0) \times B$$

$$E + B \times (Z_0 + Z_1) = B + B \times (Z_0 \times Z_0 \times Z_1)$$

$$E + B \times (Z_0 \times Z_1) = B + (Z_0 \times (Z_0 + Z_1)) \times B$$

$$B = E + Z_0 + (Z_1 + Z_0 \times Z_1) \times B$$

$$B = E + (Z_0 + Z_1) \times B$$

## Tests understanding of an important topic

### AofA Intro Q&A 1

Q. Match each “toll function” at left with the order of growth of the solution at right for the Quicksort recurrence

$$F_N = t_N + \frac{1}{N} \sum_{1 \leq k \leq N} (F_{k-1} + F_{N-k}) \quad \text{with} \quad F_0 = 0$$

$t_N$	order of growth of $F_N$
1	0
$N$	1
0	$1/N$
$N^2$	$N$
$2N + 1$	$N \lg N$
$1/N$	$N^2$
	$N^3$

**Note.** We try hard to avoid answers that depend on detailed calculations.

## “Fair” (no tricks)

### AofA Asymptotics Q&A 3

Q. Match each expression with an approximation to its value.

$1.01^{10}$

$1.05^{10}$

$1.01^{20}$

$1.01^{50}$

$1.01^{100}$

1.10102

1.10462

1.22019

1.50034

1.62889

1.64463

2.02300

2.70481

2.71828

## Teaches something

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### AofA GFs Q&A 3

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Q. Fill the circle corresponding to the value of

$$[z^n] \sum_{0 \leq k \leq n} \binom{2k}{k} \binom{2n-2k}{n-k}$$

and justify your answer.

$2^n$         $4^n$         $2^{n/2}$

It is  $[z^n] \left( \frac{1}{\sqrt{1-4z}} \right)^2$



## Summary of guidelines for preparing Questions and Answers

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**Suggestion.** Iterate with a partner (or yourself).