AC Pole Apps Q&A: Making change in Canada

Q. How many ways to make change for N cents using only nickels, dimes and quarters?


MONEY / CANADA

Canadian Pennies Are No More

Mint stops distributing coins today

By Matt Cantor, Newser User
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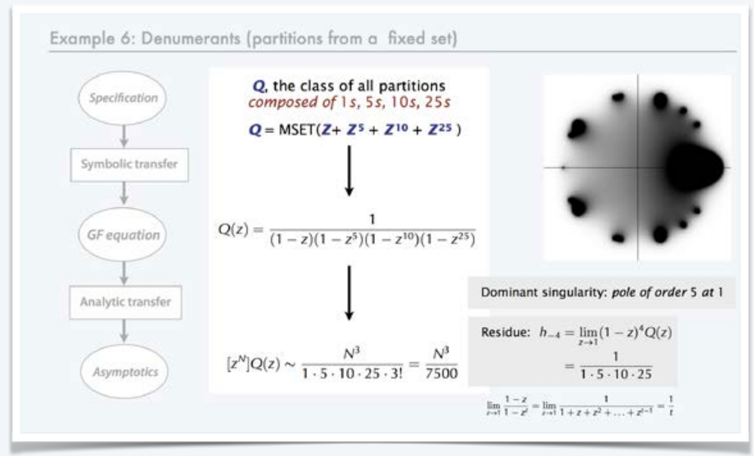
20 comments

Canada is ending the distribution of pennies. (Shutterstock)

(NEWSER) – That's it for Canada's penny: Today is the last day the coin will be distributed after minting stopped in May as a cost-saving measure. Still, there are 6 billion pennies in circulation, so they could be around for a long time: "We estimate three to four years" as they are pulled from circulation, says a Royal Canadian Mint rep. For cash shoppers, that means prices will be rounded up or down to the nearest nickel; for those paying by card, prices won't change, the *Province* reports.



$\boldsymbol{Q}$ = MSET($\boldsymbol{Z}^5 + \boldsymbol{Z}^{10} + \boldsymbol{Z}^{25}$)

$$Q(z) = \frac{1}{(1-z^5)(1-z^{10})(1-z^{25})}$$

$$[z^N]Q(z) \sim \frac{N^2}{5 \cdot 10 \cdot 25 \cdot 2!} = \frac{N^2}{2500}$$

$f(z)$ rational with a single dominant pole α

$$[z^N]f(z) = \frac{\beta^N N^{M-1}}{(M-1)!\alpha^M} \lim_{z\to\alpha} (z-\alpha)^M f(z)$$

where $\beta = 1/\alpha$ and M is the multiplicity of α

AC Pole Apps Q&A: Compositions with restrictions

Q. How many ways to write N as an ordered sum of (positive) odd integers?

1

$C_1 = 1$

1 + 1

$C_2 = 1$

1 + 1 + 1
3

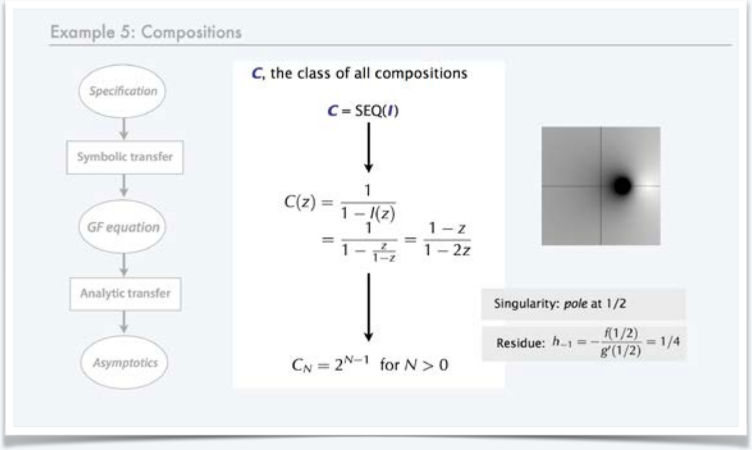
$C_3 = 2$

1 + 1 + 1 + 1
1 + 3
3 + 1

$C_4 = 3$

1 + 1 + 1 + 1 + 1
1 + 1 + 3
1 + 3 + 1
3 + 1 + 1
5

$C_5 = 5$



$$\mathbf{C} = \text{SEQ}(\,\mathbf{Z} + \mathbf{Z}^3 + \mathbf{Z}^5 + \mathbf{Z}^7 + \ldots)$$

$$C(z) = \frac{1}{1 - z - z^3 - z^5 - \ldots} = \frac{1}{1 - z\,(1 + z^2 + z^4 + \ldots)} = \frac{1}{1 - \frac{z}{1-z^2}} = \frac{1 - z^2}{1 - z - z^2}.$$

Exercise. Direct proof that it is a Fibonacci sequence ?

AC Pole Apps Q&A: "Black and white reversible strings"

Q. Asymptotics of [z^N] in $G(z) = \dfrac{z(1-3z^2)}{(1-2z)(1-2z^2)}$

A. 2^{N-2}

$f(z)$ rational with a single dominant pole α

$$[z^N]f(z) = \frac{\beta^N N^{M-1}}{(M-1)!\alpha^M} \lim_{z\to\alpha}(z-\alpha)^M f(z)$$

where $\beta = 1/\alpha$ and M is the multiplicity of α

Exercise. Prove that G(z) is the OGF for "black-and-white reversible strings"