

# AC Pole Apps Q&A: Making change in Canada

Q. How many ways to make change for  $N$  cents using only nickels, dimes and quarters?

MONEY / CANADA

## Canadian Pennies Are No More

Mint stops distributing coins today

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20 comments



(NEWSER) – That's it for Canada's penny: Today is the last day the coin will be distributed after minting stopped in May as a cost-saving measure. Still, there are 6 billion pennies in circulation, so they could be around for a long time: "We estimate three to four years" as they are pulled from circulation, says a Royal Canadian Mint rep. For cash shoppers, that means prices will be rounded up or down to the nearest nickel; for those paying by card, prices won't change, the Province reports.

Canada is ending the distribution of pennies. (Shutterstock)



Example 6: Denumerants (partitions from a fixed set)

Specification

Symbolic transfer

GF equation

Analytic transfer

Asymptotics

$Q$ , the class of all partitions composed of 1s, 5s, 10s, 25s

$Q = \text{MSET}(Z + Z^5 + Z^{10} + Z^{25})$

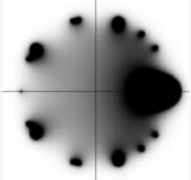
$$Q(z) = \frac{1}{(1-z)(1-z^5)(1-z^{10})(1-z^{25})}$$

$$[z^N]Q(z) \sim \frac{N^3}{1 \cdot 5 \cdot 10 \cdot 25 \cdot 3!} = \frac{N^3}{7500}$$

Dominant singularity: pole of order 5 at 1

Residue:  $h_{-4} = \lim_{z \rightarrow 1} (1-z)^4 Q(z) = \frac{1}{1 \cdot 5 \cdot 10 \cdot 25}$

$\lim_{z \rightarrow 1} \frac{1-z}{1-z^2} = \lim_{z \rightarrow 1} \frac{1}{1+z} = \frac{1}{2}$



$$Q = \text{MSET}(Z^5 + Z^{10} + Z^{25})$$

$$Q(z) = \frac{1}{(1-z^5)(1-z^{10})(1-z^{25})}$$

$$[z^N]Q(z) \sim \frac{N^2}{5 \cdot 10 \cdot 25 \cdot 2!} = \frac{N^2}{2500}$$

$f(z)$  rational with a single dominant pole  $\alpha$

$$[z^N]f(z) = \frac{\beta^N N^{M-1}}{(M-1)! \alpha^M} \lim_{z \rightarrow \alpha} (z-\alpha)^M f(z)$$

where  $\beta = 1/\alpha$  and  $M$  is the multiplicity of  $\alpha$

# AC Pole Apps Q&A: Compositions with restrictions

Q. How many ways to write  $N$  as an ordered sum of (positive) odd integers?

$1$ $C_1 = 1$	$1 + 1 + 1$ $3$ $C_3 = 2$	$1 + 1 + 1 + 1 + 1$ $1 + 1 + 3$ $1 + 3 + 1$ $3 + 1 + 1$ $5$ $C_5 = 5$
$1 + 1$ $C_2 = 1$	$1 + 1 + 1 + 1$ $1 + 3$ $3 + 1$ $C_4 = 3$	

Example 5: Compositions

Specification

Symbolic transfer

GF equation

Analytic transfer

Asymptotics

**C**, the class of all compositions

$C = \text{SEQ}(1)$

$$C(z) = \frac{1}{1 - J(z)}$$

$$= \frac{1}{1 - \frac{z}{1-z}} = \frac{1-z}{1-2z}$$

$C_N = 2^{N-1}$  for  $N > 0$

Singularity: pole at  $1/2$

Residue:  $h_{-1} = -\frac{f(1/2)}{g'(1/2)} = 1/4$

$$C = \text{SEQ}(Z + Z^3 + Z^5 + Z^7 + \dots)$$

$$C(z) = \frac{1}{1 - z - z^3 - z^5 - \dots} = \frac{1}{1 - z(1 + z^2 + z^4 + \dots)} = \frac{1}{1 - \frac{z}{1-z^2}} = \frac{1 - z^2}{1 - z - z^2}$$

Exercise. Direct proof that it is a Fibonacci sequence ?

## AC Pole Apps Q&A: “Black and white reversible strings”

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Q. Asymptotics of  $[z^N]$  in  $G(z) = \frac{z(1-3z^2)}{(1-2z)(1-2z^2)}$

A.  $2^{N-2}$

$f(z)$  rational with a single dominant pole  $\alpha$

$$[z^N]f(z) = \frac{\beta^N N^{M-1}}{(M-1)!\alpha^M} \lim_{z \rightarrow \alpha} (z - \alpha)^M f(z)$$

where  $\beta = 1/\alpha$  and  $M$  is the multiplicity of  $\alpha$

**Exercise.** Prove that  $G(z)$  is the OGF for “black-and-white reversible strings”